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Mechanism and Machine Theory 40 (2005) 821–833
Mechanism and Machine Theory
www.elsevier.com/locate/mechmt

Jaw mechanism modeling and simulation

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Received 19 May 2003; received in revised form 30 August 2004; accepted 28 December 2004
Available online 2 March 2005

Abstract

To quantitatively evaluate the dynamic changes to the texture of foods during chewing a mechatronic device is required to reproduce human chewing behaviour. To design such a device the jaw mechanism needs to be firstly modelled and analysed through simulations. Following an investigation into the biological process of mastication, the muscles responsible for the chewing movements are represented by a set of linear actuators. They are then placed between the mandible and the skull according to the biological structure and functionality, resulting in a spatial mechanism of 14 links, six linear actuators and 12 spherical joints. Simulations for motion and control have been conducted using the Matlab SimMechanics toolbox, and results have shown that the jaw mechanism enables the jaw movements to be reproduced.

Keywords: Jaw mechanism; Mastication; Food texture; Chewing robot

1. Introduction

Food preference is determined in a large part by its sensory perception [1], which includes appearance, taste, aroma and texture. In particular, texture perception is an important factor

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0094-114X/$ - see front matter
doi:10.1016/j.mechmachtheory.2004.12.011

in consumer sensory appreciation. For certain categories of foods, texture is of predominant importance. The foods include those with a bland flavor such as rice and pasta, or those possessing the characteristics of crispness and crunchiness, for example snacks, many fresh fruits and vegetables. Therefore, the development of texturally attractive food products increasingly interests food industries. To do so, it is essential to understand the relationships between food structure and texture perception and the interaction between texture perception and mastication process. The study on the relationships between food properties, mastication process and texture perception can be categorized into three main areas, sensory assessment such as time intensity (TI) [2–4] or progressive profiling [5], physiological study such as electromyography (EMG) recording [6–9] or jaw movement recording [10,11], and mechanical measurement [12] such as compression or shear test of food. Sensory panels may only provide qualitative feedback while instrumental measurements are too abstract for characterising the dynamic interactions between texture perception and chewing patterns that occur during the process of food structure breakdown. The physiological method has not been used for food evaluation due to sophisticated mastication process and mechanism of texture perception.

Several chewing simulation devices have been developed for different purposes [13–17]. One was used for testing dental materials [13], in which two servo-hydraulic actuators are combined to produce the force movement cycle of human mastication. The device, though good enough for dental materials testing, cannot be used to reproduce the complex chewing movements in three-dimensional space. A jaw simulator (JSN/2A) developed at Niigata University, Japan was developed to clarify the control mechanism of jaw movement [14,15]. It has an upper and a lower jaw equipped with tooth-contact and bite-force sensors and a condylar housing for the temporo-mandibular joint (TMJ). The simulator is actuated by a number of wire-tendon DC-servo actuators for simulating dominant masticatory muscles. A mastication robot developed by Waseda University, Japan was to investigate various aspects of human chewing behaviour [16,17]. The latest version of the robot (WJ-3) has three degrees of freedom (DOF), nine artificial muscle actuators (AMA) and two micro pressure sensors on each of the lower and the upper molar teeth for measuring the biting forces when chewing foods. Since the mandible movements are in three-dimensional space and require at least six DOFs, the WJ-3 robot still cannot be used to reproduce complex human chewing behaviours. The Rosy robot system was used to measure the jaw movements of six DOFs that were visualised through a simulator for functional diagnostics [18]. However, this system was not supposed to perform chewing.

The ultimate aim of this study is to develop a mechatronic machine that can reproduce the jaw movements with motion and force requirements specified according to human chewing behaviour. The machine is intended for evaluation of new types of foods in terms of food texture perception. This paper presents a jaw mechanism model and some simulation results of the jaw motion under open- and closed-loop controls. The muscles responsible for the chewing movements are firstly represented by a set of linear actuators, which are then placed between the mandible (or the end-effector) and the skull (or the ground) according to the biological structure and functionality. This arrangement ends up with a spatial mechanism. Simulations have been conducted using the Matlab SimMechanics toolbox, and results have shown that the jaw mechanism is able to reproduce the jaw movements and that the highly dynamically coupled jaw movements require a sophisticated control system.

2. Jaw muscles and movements

More than 20 muscles are involved in the process of mastication [19,20]. The temporalis muscle, as shown in Fig. 1(a), is a large, flat muscle. Its fibres can be divided into two parts: the anterior fibres that elevate the mandible (lower jaw) and close the mouth and the posterior fibres which contribute to the complex grinding movement by retracting the mandible. The pterygoid (Fig. 1(b)) are a family of muscles: lateral and medial pterygoids. The lateral pterygoids work to protract the mandible and open the mouth, and medial pterygoids mostly protracts the mandible. The masseter, as shown in Fig. 1(c), is a flat quadrilateral muscle with deep and superficial parts. It contributes mostly to the mandible elevation (mouth closing), and also plays a role in protracting the mandible. Underneath the mandible, the hyoid bone supports a set of muscles called suprahyoid muscles (Fig. 1(d)). Among them, digastric, stylohyoid, mylohyoid, geniohyoid and platysma muscles are involved in the mouth opening and then the depression of the mandible.

The mandible, or the lower jaw, is attached to the rest of the skull by muscles through a so-called temporo-mandibular joint, as shown in Fig. 1(b). Thus, it cannot move as a free body in space as it is constrained by biological joints and muscles. Human chewing behaviour can be described by two basic movements of the mandible: the clenching and the grinding movements (Fig. 2). Clenching consists of the successive elevation and depression of the mandible and uses a lot of

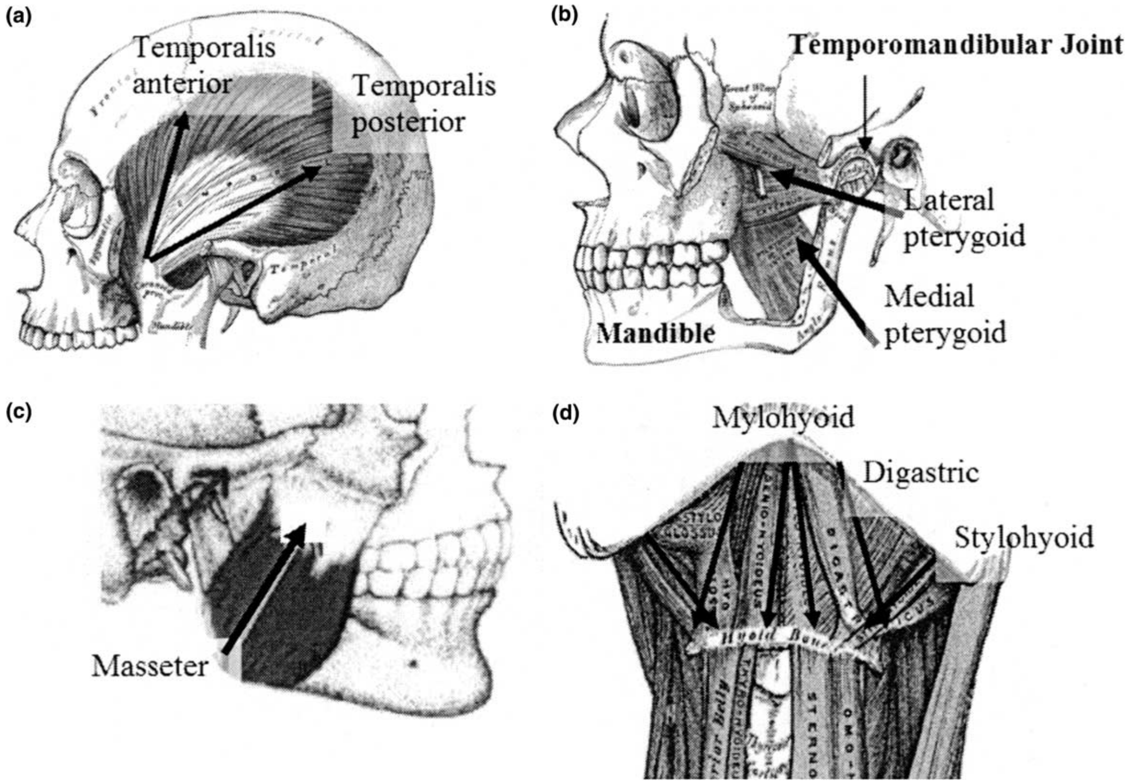

Fig. 1. Muscles for mastication (reconstructed after [19,20]): (a) left temporalis muscles, (b) left pterygoids muscles, (c) right masseter and (d) suprahyoid muscles.

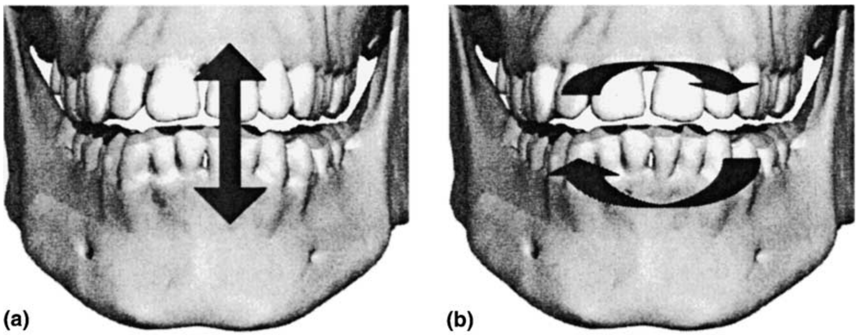

Fig. 2. Two basic chewing movements: (a) basic clenching and (b) basic grinding.

muscles but especially the masseter and temporalis anterior. Grinding involves almost all the jaw muscles and the incisal point (the point between the two lower incisives) traces a circle in the horizontal plane. Thus, a complex human mastication can be regarded as aggregate clenching and grinding movements.

3. Jaw mechanism modelling

3.1. Mandible reference points

Reproducing the jaw movements in three-dimensional space requires an appropriate mechanism. The movability of a mandible can be observed by trajectories of a few reference points. Shown in Fig. 3 are the reference points commonly observed: incisal point (IP), kinematic condylar point (i.e., LCP and RCP) and first molar point (i.e., LMP and RMP) [21]. The incisal point is the point between the two lower jaw incisives. The kinematic condylar points are the points on the top of left and right processus, simply called left processus and right processus. The two points are very constrained because they have to stay in the mandibular fossa. The first molar points are those on the first lower molars. The trajectories of these five reference points constitute qualitative specifications that should be achieved by the jaw mechanism to be developed.

3.2. Jaw mechanism model

A muscle works along a linear axis and actuates only in contraction phase. Linear actuators are used to replace muscles in provision of both contraction and depression action in this study. As discussed previously, the muscles collectively generate the movement of the mandible pivoted at temporo-mandibular joints, and the mandible movement is regarded as an aggregate clenching and grinding movement. For the clenching the anterior temporalis and masseter muscles are responsible for a large measure of the mandible elevation whereas the suprahyoid muscles and gravity perform the depression. Therefore, four linear actuators are used to produce the mandible elevation and depression in place of anterior temporalis, masseter and suprahyoid muscles. They are called right masseter, left masseter, right temporalis and left temporalis actuators, as shown in Fig. 4. For the grinding movement which is mainly produced by the left and right posterior tem-

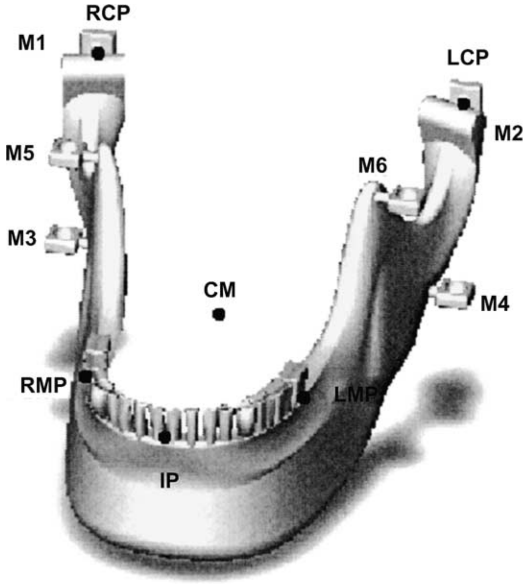

Fig. 3. A mandible model and the reference points. IP stands for incisal point, LCP for the left condylar point and RCP for the right condylar point.

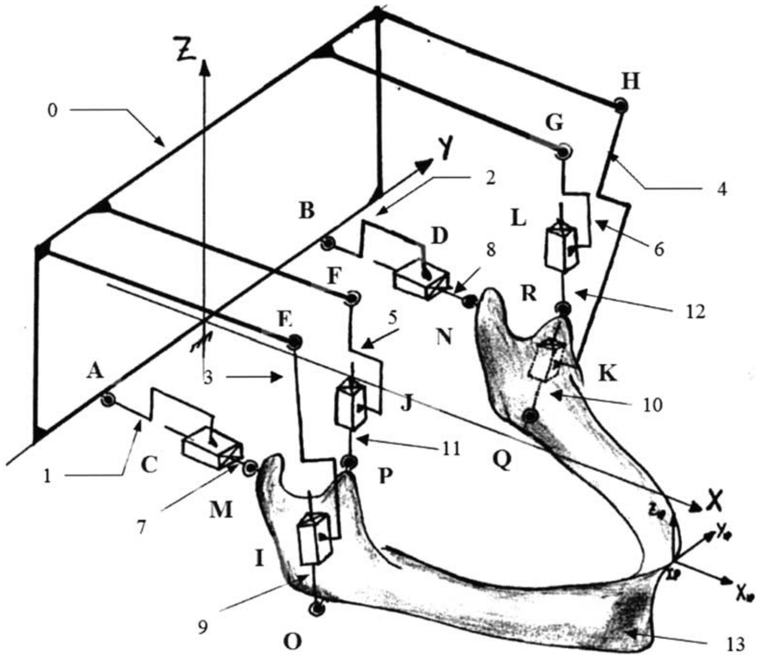

Fig. 4. A kinematic sketch of jaw mechanism. Letters C, D, I, J, K, L stand for muscular actuators, the numbers for links making up the mechanism and the remaining letters for actuators ground and mandible attaching points (or spherical joints).

Table 1
Notations of jaw mechanism constituents

Links	Passive joints	Active joints (actuators)
0: Skull (ground)	A: back right mandibular fossa	C: right grinder
1: Back right grinder	B: back left mandibular fossa	D: left grinder
2: Back left grinder	E: upper right masseter attach	I: right masseter
3: Upper right masseter	F: upper right temporalis attach	J: right temporalis
4: Upper left masseter	G: upper left temporalis attach	K: left temporalis
5: Upper right temporalis	H: upper left masseter attach	L: left masseter
6: Upper left temporalis	M: frontal right mandibular fossa	
7: Frontal right grinder	N: frontal left mandibular fossa	
8: Frontal left grinder	O: lower right masseter attach	
9: Lower right masseter	P: lower right temporalis attach	
10: Lower left masseter	Q: lower left temporalis attach	
11: Lower right temporalis	R: lower left masseter attach	
12: Lower left temporalis		
13: Mandible (end-effector)		

poralis, two actuators are placed behind each mandible processus connecting to the temporomandibular joint. They are called right grinder and left grinder (Fig. 4). Each actuator is then attached to the skull and mandible through spherical joints at the two ends, resulting in the jaw mechanism model (Fig. 4). The spatial model consists of 14 links, six prismatic actuators and 12 spherical joints, as given in Table 1.

It is essential to check whether or not the reproduction of human-like chewing movements is achievable with this model in terms of kinematics. This is concerned with the mechanism mobility validity that can be evaluated using the Kutzbach criterion [22],

$$m = 6(n-1) - 5j1 - 4j2 - 3j3 - 2j4 - j5 \tag{1}$$

where m is the number of independent DOFs required for the model being a movable mechanism, n (=14) the number of links, $j1$ (=6), $j2$ (=0), $J3$ (=12), $j4$ (=0) and $j5$ (=0) the numbers of the 1-, 2-, 3-, 4- and 5-DOF joints, respectively. The calculated number of independent DOFs is $m = 6 \times 13 - 5 \times 6 - 3 \times 12 = 12$. Considering there is one local rotational movement associated with each actuator (i.e., between each pair of spherical joints), the final independent DOFs becomes $m = 12 - 6 = 6$. This number is the number of active joints (actuators), thus proving that the reproduction of the chewing movements using the mechanism model is possible.

4. Simulations

To analyse the mandible movements given actuation inputs or to find suitable actuation inputs (controls) for a predefined mandible motion, SimMechanics, one of the toolboxes for Matlab Release 13 was used [23]. The toolbox is an object-oriented program able to perform simulations of most motion analysis problems of machines without a need of dynamic equations. The physical assembly of a machine and its physical quantities as well as actuation/control mode must be provided (see Fig. 5).

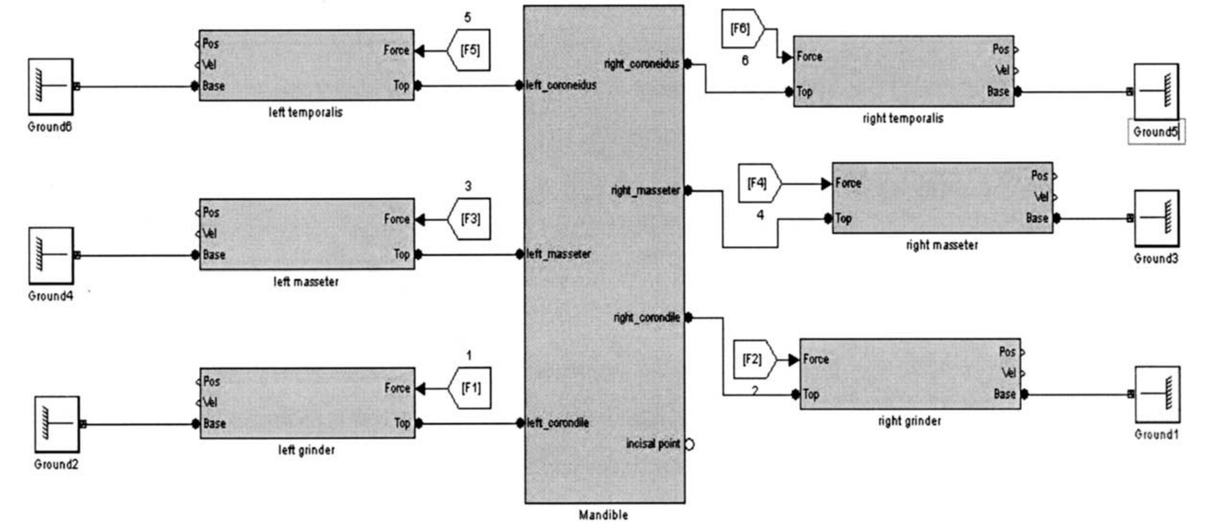

Fig. 5. Screenshot of the jaw mechanism in SimMechanics. The middle block stands for the mandible, the most left and right blocks for the ground and the blocks between the mandible and ground for muscular actuators.

4.1. Approximation and physical quantities

To compute a machine model in SimMechanics, all physical bodies need to be specified in terms of inertial matrix expressed at their gravity centers. The mandible, which has a complicated shape, is approximated by a set of five parallelopipeds rigidly welded, and the actuators were regarded as two steel cylinders linked by a cylindrical joint.

The approximated jaw mechanism has three types of bodies: the skull (or the ground), the actuators and the mandible (or the end-effector). Their dimensions and mass properties were measured, estimated or calculated from a replica skull. The skull, as the ground of the jaw mechanism does not need the mass property specified. Tables 2 and 3 give the coordinates of the points (G_i, $i = 1,2,\ldots,6$) where the actuators are attached to the ground and the coordinates of the points (M_i, $i = 1,2,\ldots,6$) where the actuators to the mandible, respectively.

Each actuator is composed of one hollow and one solid cylinder, its total length is the distance between the attachment points M_i and G_i. It is assumed that each cylinder is 3/4 of the total length

Table 2
Ground attaching point coordinates of the mechanism[a]

Description	Origin	X_0 (m)	Y_0 (m)	Z_0 (m)
Attaching the left grinder to the skull	Gl	−0.04	0.05	−0.01
Attaching the right grinder to the skull	G2	−0.04	−0.05	−0.01
Attaching the left masseter to the skull	G3	0.015	0.065	−0.015
Attaching the right masseter to the skull	G4	0.015	−0.065	−0.015
Attaching the left temporalis to the skull	G5	0.04	0.05	0.075
Attaching the right temporalis to the skull	G6	0.04	−0.05	0.075

[a] All coordinates are defined in the $(0, X_0, Y_0, Z_0)$ pair, also called world coordinates system (WCS).

Table 3
End-effector attaching point coordinates of the mechanism

Description	Origin	X_0 (m)	Y_0 (m)	Z_0 (m)
Attaching the left grinder to the mandible	Ml	0	0.05	−0.01
Attaching the right grinder to the mandible	M2	0	−0.05	−0.01
Attaching the left masseter to the mandible	M3	0.015	0.055	−0.095
Attaching the right masseter to the mandible	M4	0.015	−0.055	−0.095
Attaching the left temporalis to the mandible	M5	0.03	0.05	−0.005
Attaching the right temporalis to the mandible	M6	0.03	−0.05	−0.005

Table 4
Actuators properties[a]

Actuator name	Nominal length (m)	Mass of hollow cylinder (kg)	Mass of solid cylinder (kg)	Total mass (kg)	Stroke (m)
Right grinder	0.04	0.219	0.073	0.292	±0.01
Left grinder	0.04	0.219	0.073	0.292	±0.01
Right masseter	0.0806	0.442	0.147	0.589	±0.02
Left masseter	0.0806	0.442	0.147	0.589	±0.02
Right temporalis	0.0806	0.442	0.147	0.589	±0.02
Left temporalis	0.0806	0.442	0.147	0.589	±0.02

[a] Steel density is 7747 kg/m^3.

Table 5
Mandible dimensions and mass properties[a]

Mandible components	GC	GC position in WCS	Length (m)	Width (m)	Height (m)	Mass (kg)
Right processus	Al	[0.015 −0.05 −0.05]	0.03	0.01	0.1	0.027
Right side	A2	[0.07 −0.05 −0.09]	0.07	0.01	0.02	0.0126
Central part	A3	[0.095 0 −0.09]	0.01	0.09	0.02	0.0162
Left side	A4	[0.07 0.05 −0.09]	0.07	0.01	0.02	0.0126
Left processus	A5	[0.015 0.05 −0.05]	0.03	0.01	0.1	0.027

[a] Bone density is estimated at 900 kg/m^3.

long, the hollow cylinder is 1 cm in inner radius and 2 cm in outer radius, the solid cylinder is 1 cm in radius, and actuator stroke is chosen of ±1/4 of its length. Table 4 presents the physical properties of the six actuators.

Five parallelepipeds make up the mandible and a parallelepiped is defined by its length (along X-axis), its width (along Y-axis) and its height (along Z-axis). Table 5 gives the physical properties of the mandible.

4.2. Actuation modes

The jaw mechanism in SimMechatronics is illustrated in Fig. 4, where the mandible is moved by the six actuators placed between the mandible and the skull (or ground). Two actuating modes are available in SimMechanics, one for specifying pure motion of the actuators and the other for applying a force to the actuators. The former is used to perform an open-loop simulation (mainly to find the trajectories of some reference points on the mandible) while the latter is to control the jaw mechanism to achieve some prespecified mandible movement and chewing forces in closed-loop manner. Both of the open- and closed-loop simulations of the mechanism were conducted. For the open-loop simulations the mandible motion was specified in terms of its position and orientation and the actuations were calculated via a Simulink generator which was originally developed for the Stewart platform mechanism [24]. The inputs of this generator are three time-functions of the mandible position in the *X*-, *Y*- and *Z*-axis, respectively and three time-functions of the mandible orientation, defined as the angles around the *X*-, *Y*- and *Z*-axis, respectively. The outputs are the position, velocity and acceleration of all actuators.

The closed-loop simulations require designing a controller for each of the six actuators, which evaluates the current state of the jaw mechanism and then calculates the forces being applied onto each actuator. While the real mastication process involves application of biting forces on foods, the control of the pure mandible motion without taking into account the forces was considered at

Table 6
Desired chewing movement

Generator input	Symbol	Signal type	Time-function
Roll angle of the mandible	Xang	Constant	0
Pitch angle of the mandible	Yang	Sinusoid	$15 + 15\sin(2\pi t)$
Yaw angle of the mandible	Zang	Sinusoid	$5\sin(2\pi t + \pi/3)$
X position of the mandible	Xpos	Sinusoid	$0.005\sin(2\pi t + \pi/3)$
Y position of the mandible	Ypos	Constant	0
Z position of the manidble	Zpos	Constant	0

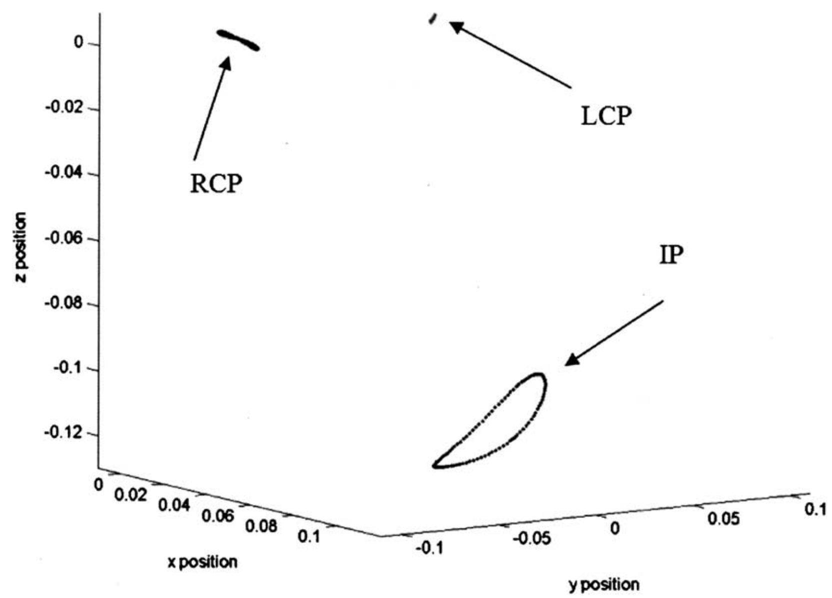

Fig. 6. Trajectories of the jaw reference points IP, LCP and RCP under open-loop control.

this stage of research. In the simulations each actuator was controlled by a PID controller independently. The desired position was extracted from the actuator generator and the difference between the desired and measured positions constitutes the error to be corrected. The error forms proportional and integral actions and the measured velocity is to generate the derivative part of the applied force.

5. Results and analysis

The desired chewing movements used in both open- and closed-loop simulations were specified by the position and orientation of the mandible given in Table 6. The chosen movement represents a banana-shaped trajectory of the incisal point [16] and a reduced range of the movements of the kinematic condylar points as these points must stay within the mandible fossa.

Fig. 6 shows the trajectories of the mandible reference points under open-loop control. It is found that the jaw mechanism reproduces a promising mandible motion in terms of the shape of the trajectories. No more quantitative analysis was made because the human chewing data

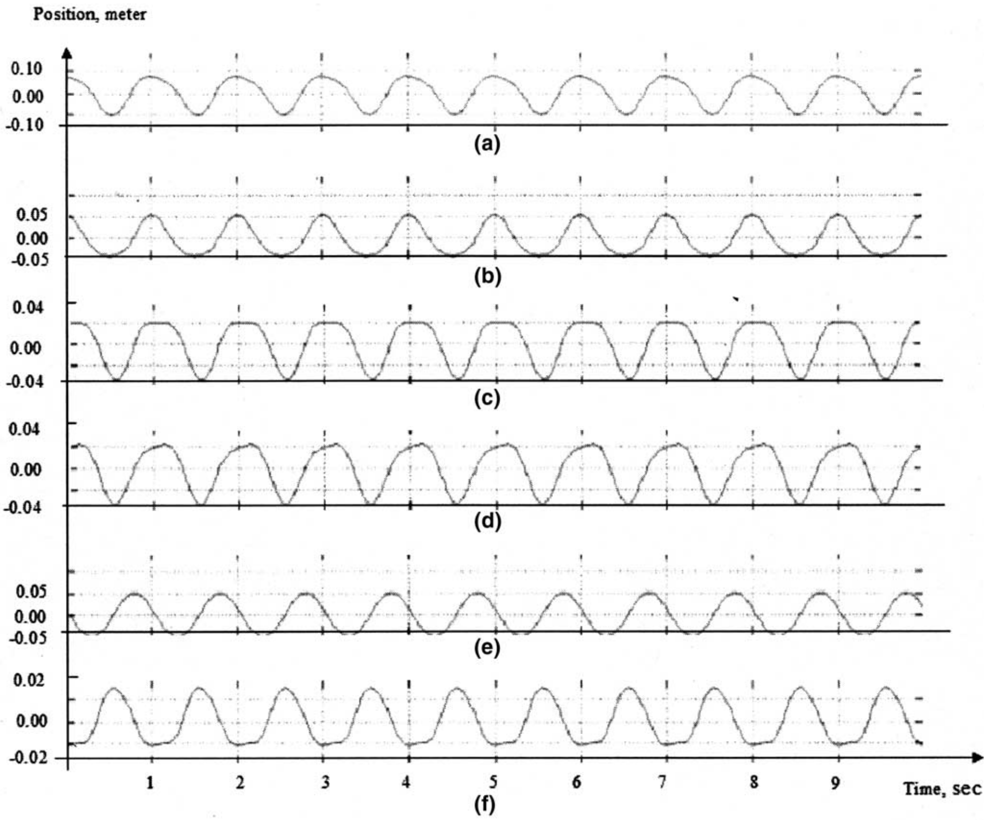

Fig. 7. Time variations of six muscular actuations: (a) right grinder, (b) left grinder, (c) right masseter, (d) left masseter and (e) right temporalis.

of various food samples had not been made available at this stage of the project research. Fig. 7 presents the time variations of the six actuations which were found from inverse kinematics for the Stewart platform mechanism [24].

For the closed-loop simulations various sets of the PID controller gains were tried. There was no significant difference in the trajectory tracking performances of the jaw mechanism under

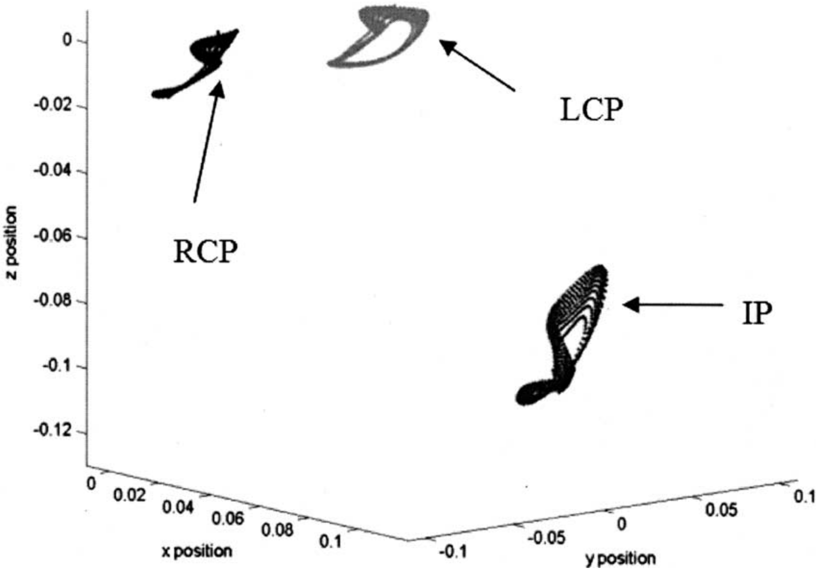

Fig. 8. Trajectories of the reference points IP, LCP and RCP under closed-loop control.

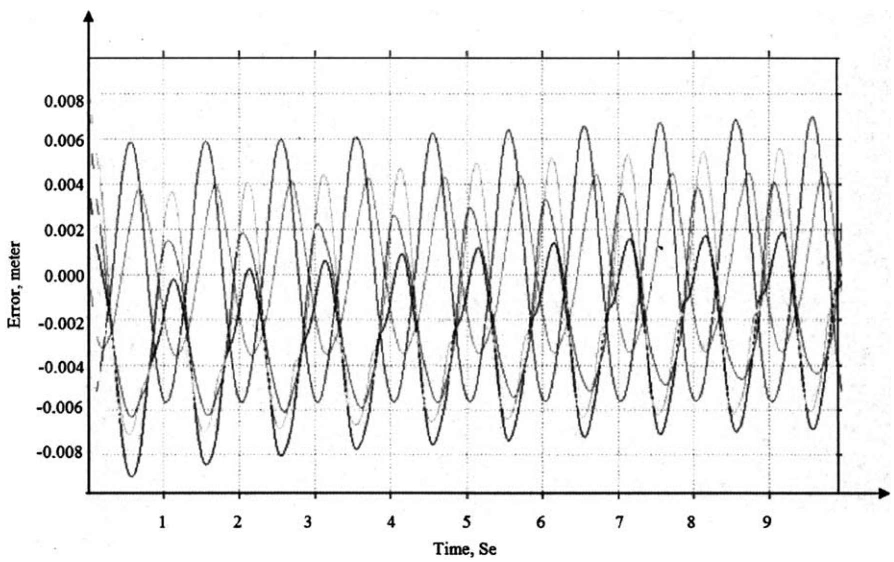

Fig. 9. Position errors in the six actuations.

control. Fig. 8 shows the trajectories of the three reference points with the gains $Kp = 2000$, $Ki = 200$ and $Kd = 200$ for the six independent PID controllers. It can be seen that the reproduced trajectories do not converge asymptotically to the desired ones and the jaw mechanism is unstable actually. This finding was further indicated by the errors in actuating positions (Fig. 9). They should vanish as time goes on. Although the trajectory tracking performance might be improved through carefully tuning the three PID gains, the six independent PID controllers could never work well for this control problem. This is because the jaw mechanism is highly coupled, nonlinear, multi-input and multi-output system, and thus, further work must be done in modeling, stability analysis, and design of a multivariable controller before coming up with a proper controller that satisfies a specified trajectory tracking performance.

6. Conclusions

A jaw mechanism model is presented in this paper. It was developed following the investigations into the biological process of mastication. The muscles responsible for the jaw chewing movements were modelled by six linear actuators. Their placement between the mandible (or the end-effector) and the skull (or the ground) resulted in a spatial mechanism of 14 links, six flinear actuators and 12 spherical joints.

Open-loop simulations have shown that the jaw mechanism was modelled properly that enables the jaw movements to be reproduced. Closed-loop simulations have shown that the jaw movements are highly dynamically coupled and their tracking is hardly achievable using a set of simple independent controllers. In order to get better trajectory tracking performance, a multivariable control should be pursued. To do this, a linear time-invariant model of the mechanism dynamics must be extracted.

While this work presented in this paper is preliminary, it does provide a viable jaw mechanism model to be worked on. The issues that are being researched currently under the project include solid model and animation, physical mechanism building, motion control and design, dynamics and multivariable control of the jaw mechanism.

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